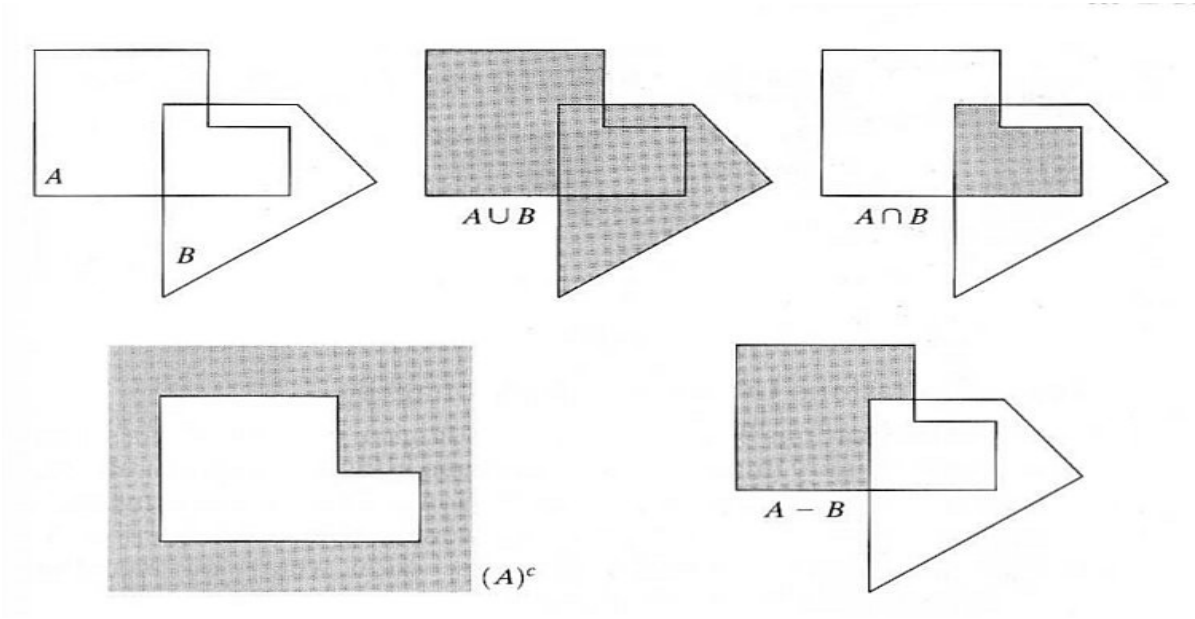
EDGE DETECTION AND SEGMENTATION OF IMAGES, II

- Mathematical morphology
- Erosion
- Dilation
- Opening and closing
- Gray-scale morphology
- Erosion and dilation of gray-scale images
- Opening and closing of gray-scale images
- Morphologic smoothing
- Morphologic gradient
- Top-hat and bottom-hat transformations
- Boundary extraction
- Top-hat transformation and boundary extraction

Mathematical morphology

- Mathematical morphology is a tool for extracting image components that are useful in the representation and description of region shape, such as boundaries, skeletons, and the convex hull
- Morphological techniques for pre- or post-processing , such as morphological filtering, thinning, and pruning
- The language of mathematical morphology is set theory



Mathematical morphology

- In addition to the basic set definitions
- The reflection of a set B is defined as:

$$\hat{B} = \{ w \mid w = -b, \text{ for } b \in B \}$$

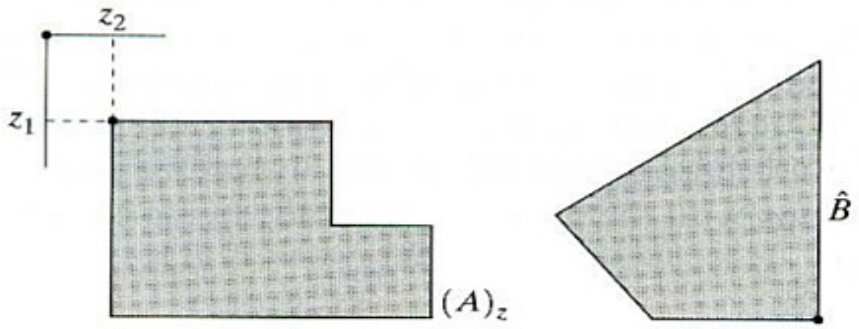
(coordinates (x, y) have been replaced by $(-x, -y)$)

- The translation of a set A by shift $z = (z_1, z_2)$ is defined as:

$$(A)_z = \{ c \mid c = a + z, \text{ for } a \in A \}$$

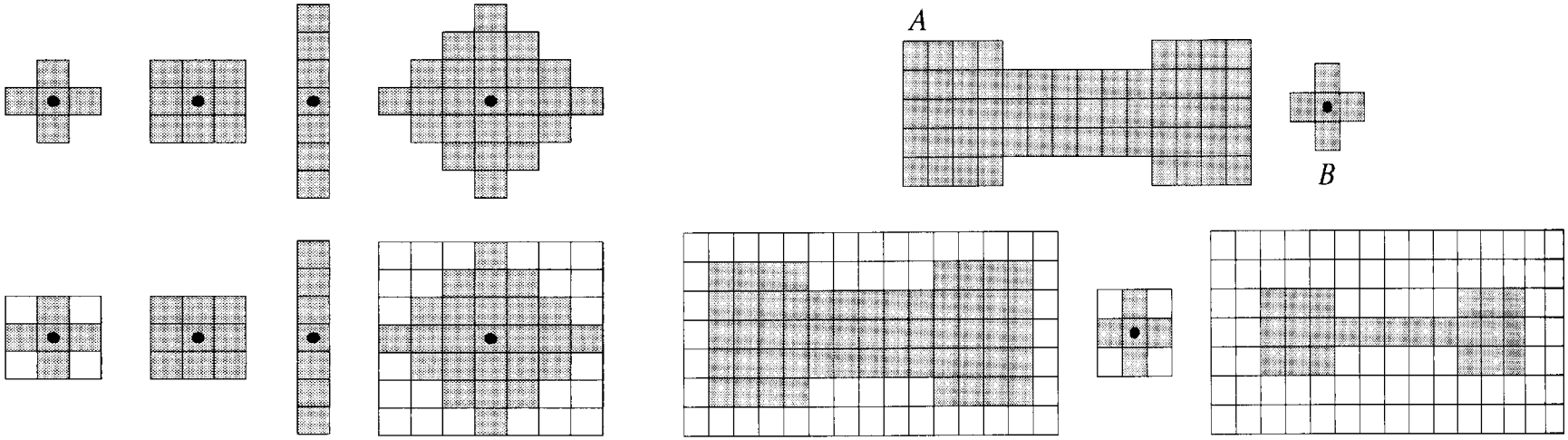
(coordinates (x,y) have been replaced by $(x+z_1, y+z_2)$)

(Gonzales, Woods)



Mathematical morphology

- Structuring elements are small sets or sub-images used to probe an image under study for properties of interest
  - Structuring elements and structuring elements converted to rectangular arrays



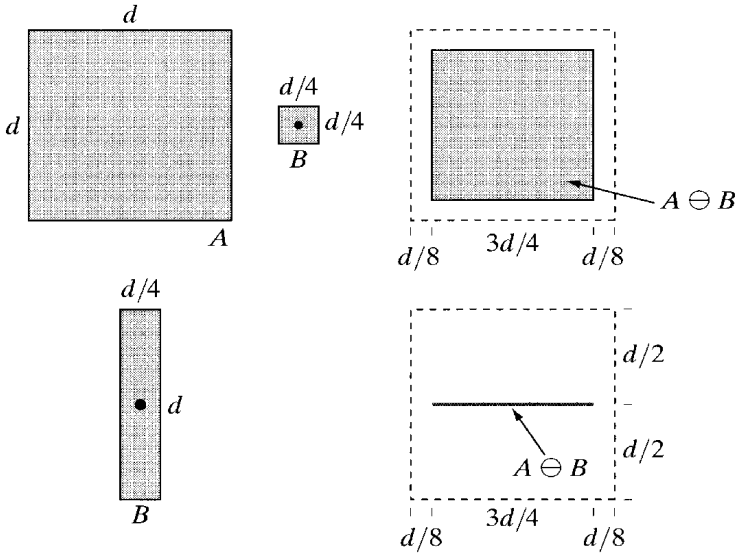
(Gonzales, Woods)

Erosion

- The erosion of A by B is the set of all points z such that B, translated by z, is contained in A

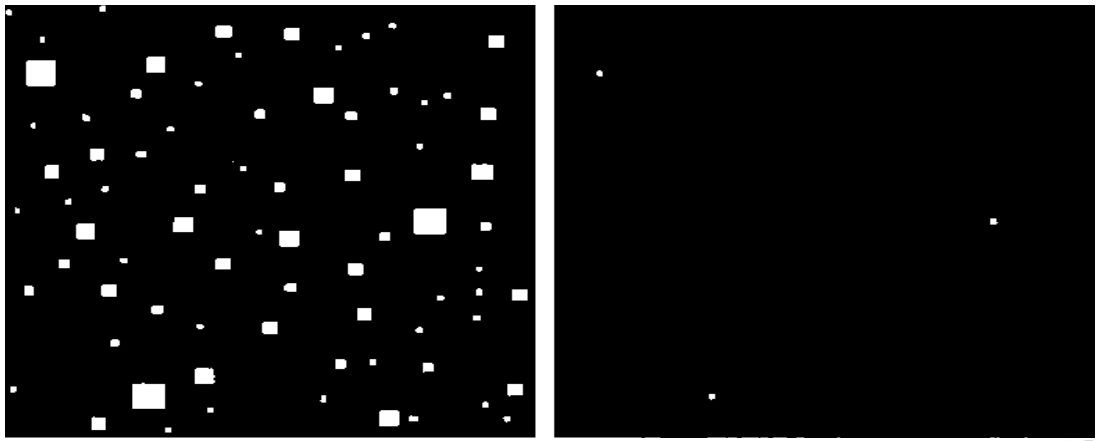
$$A \ominus B = \{z | (B)_z \subseteq A\}$$

$$A \ominus B = \{z | (B)_z \cap A^c = \varnothing\}$$



Squares of size 1, 3, 5, 7, 9, and 15 pixels

After erosion of a square structuring element of size of 13 pixels



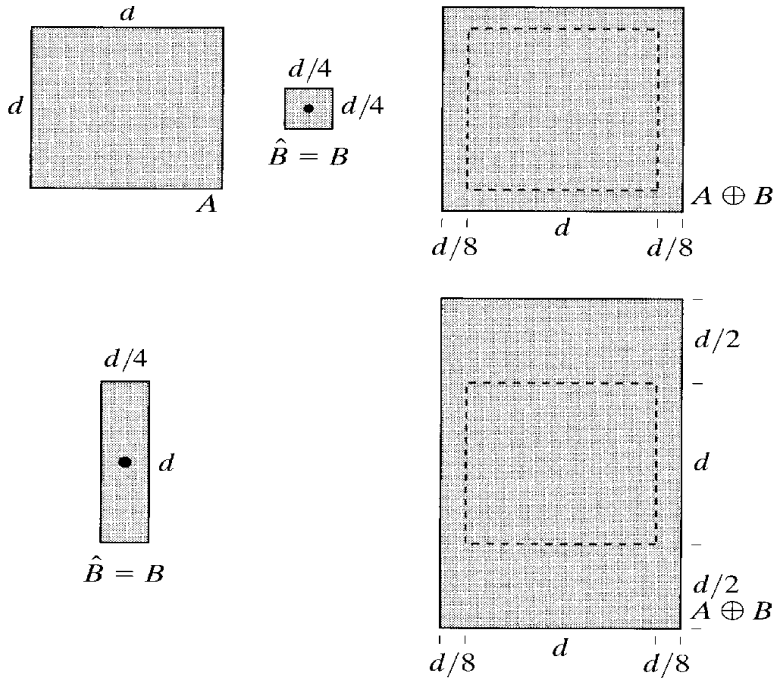
(Gonzales, Woods)

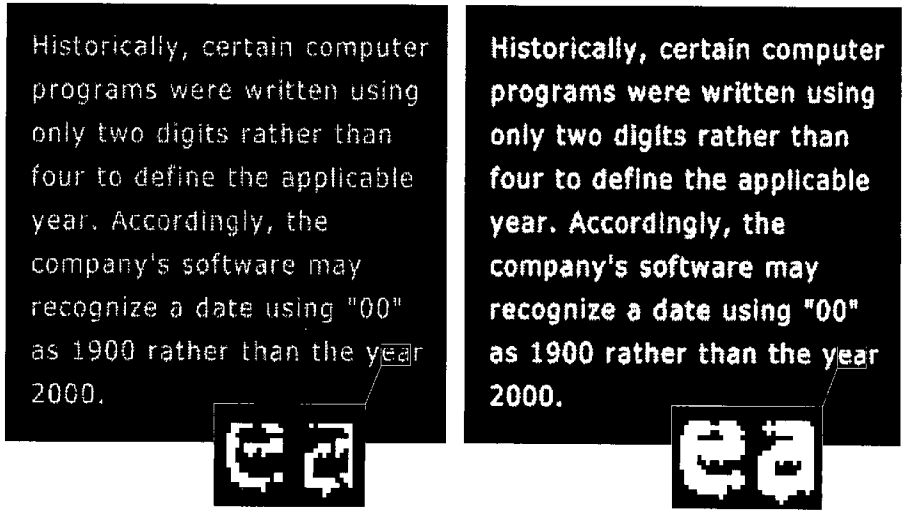
Dilation

- The dilation of A by B then is the set of all displacements, z, such that $\hat{B}$ and A overlap by at least one element B

$$A \oplus B = \left\{ z | (\hat{B})_z \cap A \neq \varnothing \right\}$$

$$A \oplus B = \left\{ z | [(\hat{B})_z \cap A] \subseteq A \right\}$$





0	1	0
1	1	1
0	1	0

(Gonzales, Woods)

Opening and closing

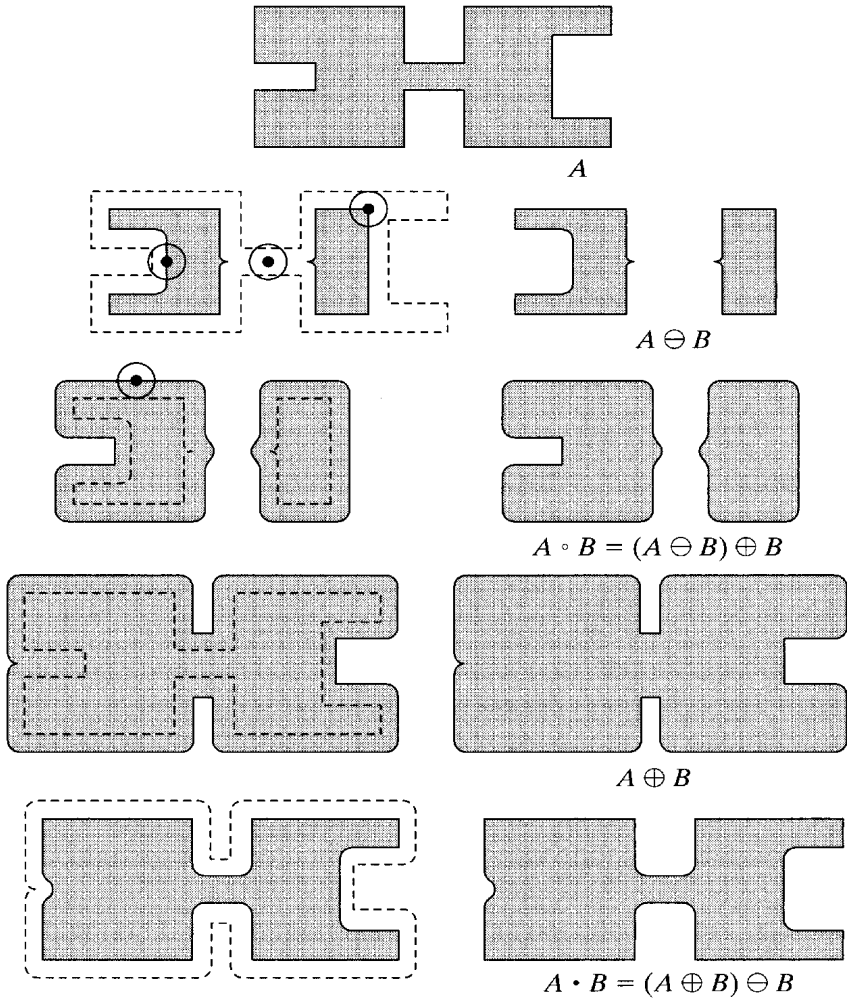
- The opening generally smooths the contour of an object, breaks narrow isthmuses, and eliminates thin protrusions

$$A \circ B = (A \ominus B) \oplus B$$

- The closing also tends to smooth sections of contours but, as opposite to opening, it generally fuses narrow breaks and long gulfs, eliminate small holes, and fills gaps in the contour

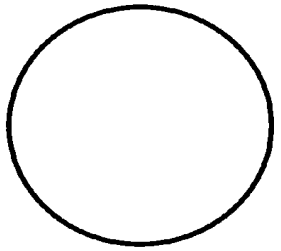
$$A \bullet B = (A \oplus B) \ominus B$$

(Gonzales, Woods)

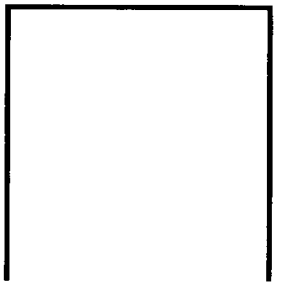


Gray-scale morphology

- The basic operations of erosion, dilation, opening, and closing can be used to process gray-scale images
- Structuring elements in gray-scale morphology belong to one of two categories: *nonflat* and *flat*
- They are used as “probes” to examine a given image for specific properties
- Flat structuring element (SE)



Flat SE



Intensity profile

(Gonzales, Woods)

Erosion and dilation of gray-scale images

- The erosion of an image f by a flat structuring element b at any location (x,y) is defined as the *minimum* value of the image in the region coincident with b when the origin of b is at (x,y)

$$[f \ominus b](x, y) = \min_{(s,t)\in b}\{f(x + s, y + t)\}$$

- To find the erosion of f by b, similar to spatial *correlation*, we place the origin of the structuring element to every pixel location in the image
- The dilation of an image f by a flat structuring element b at any location (x,y) is defined as the *maximum* value of the image in the window outlined by $\hat{b}$ when the origin of $\hat{b}$ is at (x,y)

$$[f \oplus b](x, y) = \max_{(s,t)\in b}\{f(x - s, y - t)\}$$

- To find the dilation of f by b, similar to spatial *convolution*, we place the origin of the reflected structuring element to every pixel location in the image

Opening and closing of gray-scale images

- The opening of an image f by structuring element b at any location (x,y) is:

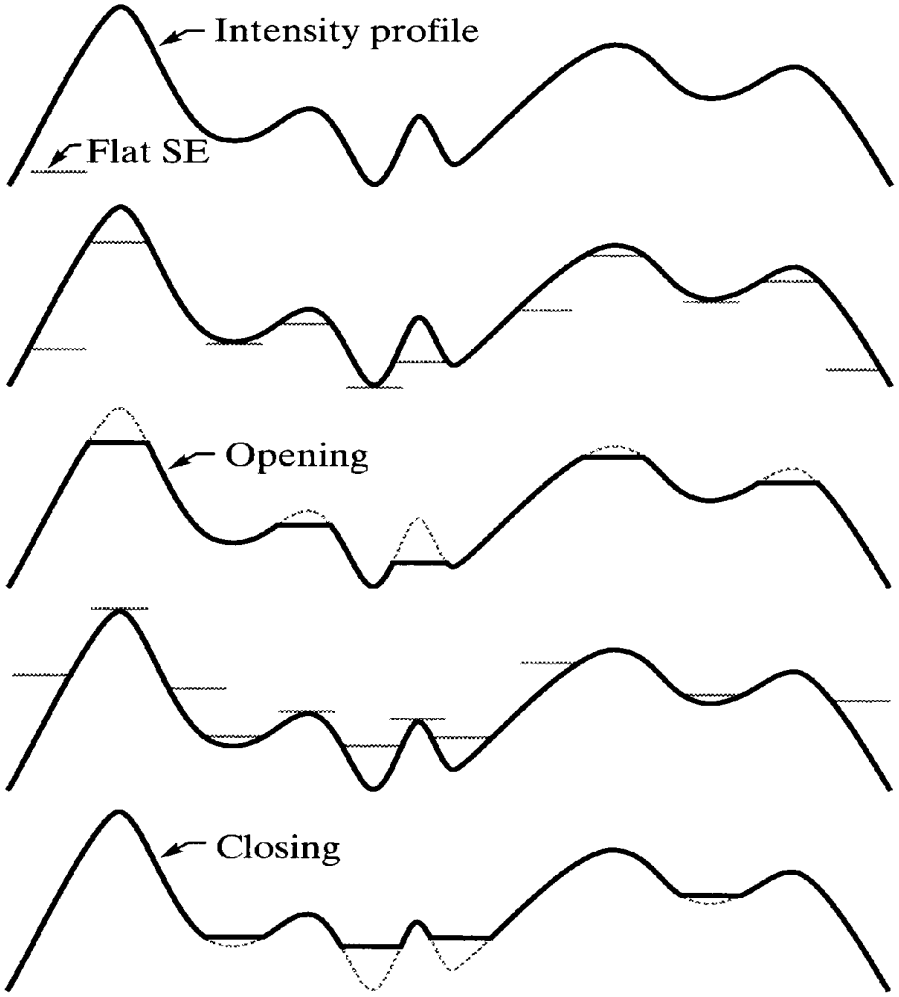
$$f \circ b = (f \ominus b) \oplus b$$

- Opening is simply the erosion of f by b, followed by a dilation of the result with b
- The closing of f by structuring element b at any location (x,y) is

$$f \bullet b = (f \oplus b) \ominus b$$

- Closing is simply the dilation of f by b, followed by an erosion of the result with b

Opening and closing of gray-scale images

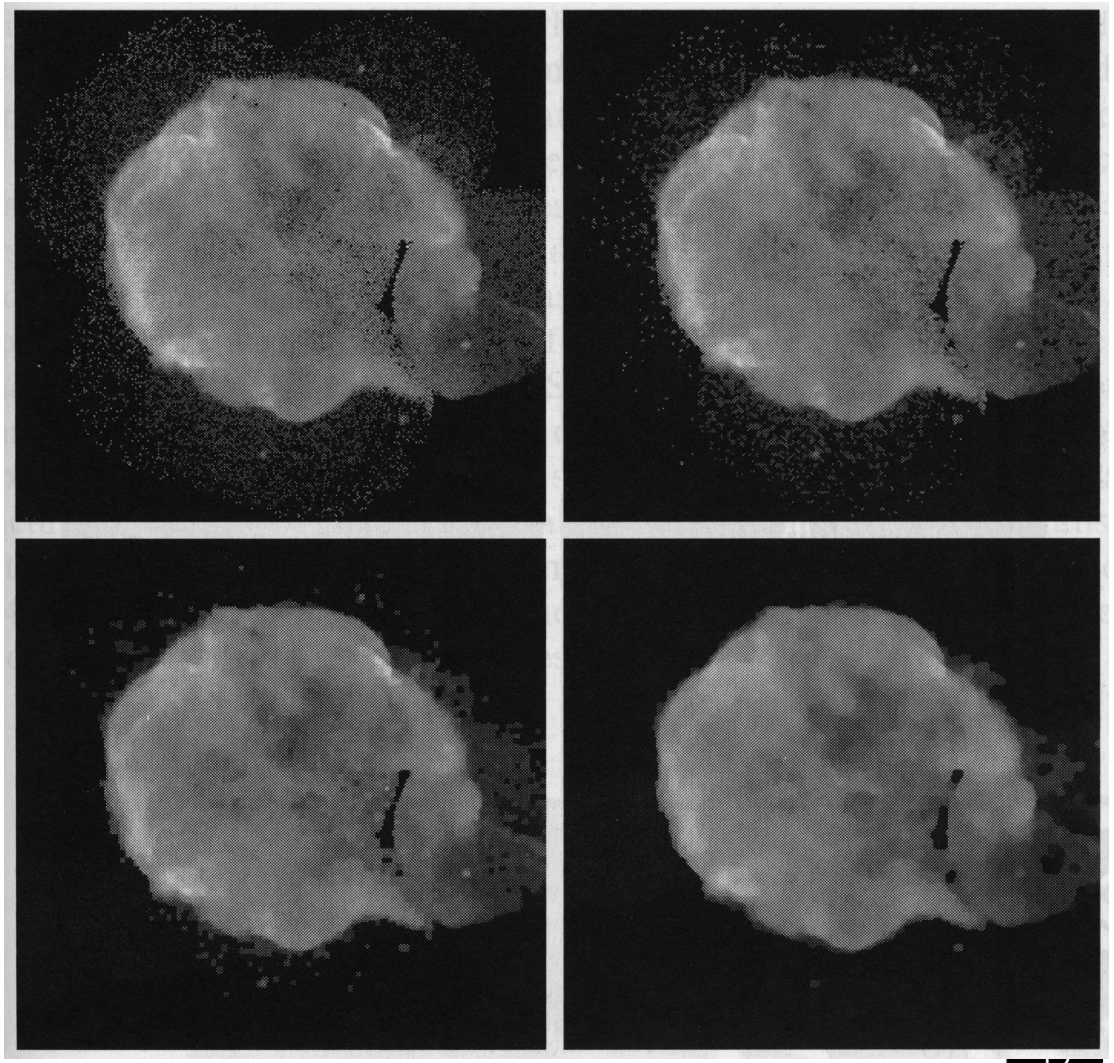


(Gonzales, Woods)

Morphologic smoothing

- Morphologic smoothing is performed by opening the original image with a structuring element (disk of given radii (1, 3, 5)) and then closing the opening with a structuring element of the same size →

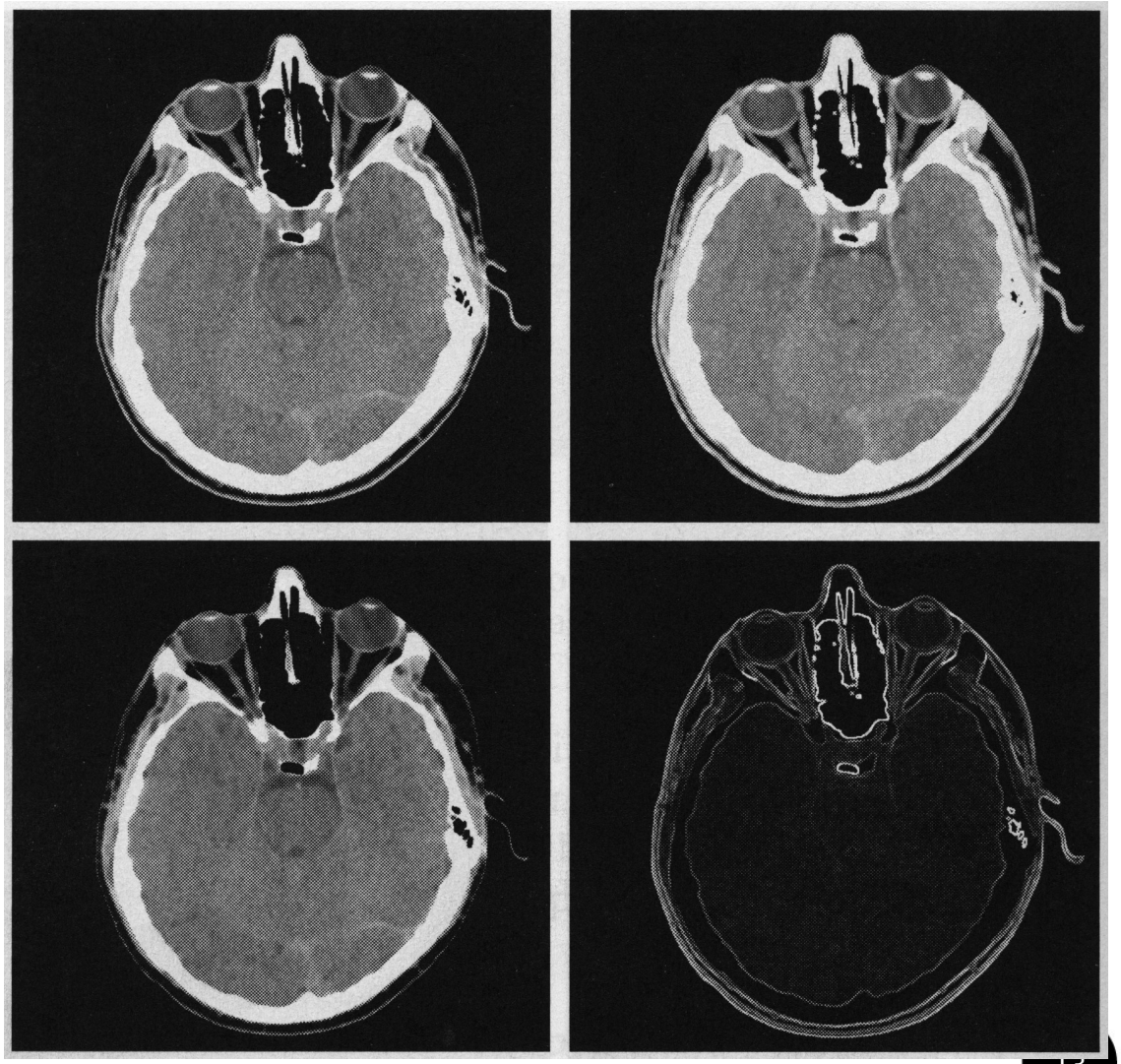
- Morphologic smoothing is performed by alternating sequential filtering in which the opening-closing sequence starts with original image, but subsequent steps perform the opening and closing on the results of the previous step



(Gonzales, Woods)

Morphologic gradient

- Objective: to extract the edges of the outer contour of the brain (the gray region), the contour of the spinal region (directly behind the nose, toward the front of the brain), and the outer contour of the head
- Objective: to generate the thinnest, continuous contours possible, while eliminating edge details related to the gray content in the eyes and brain areas



(Gonzales, Woods)

Morphologic gradient

- Morphologic gradient of an image can be obtained by subtraction of dilation of the image and erosion of the image:

$$g = (f \oplus b) - (f \ominus b)$$

(a) (b)

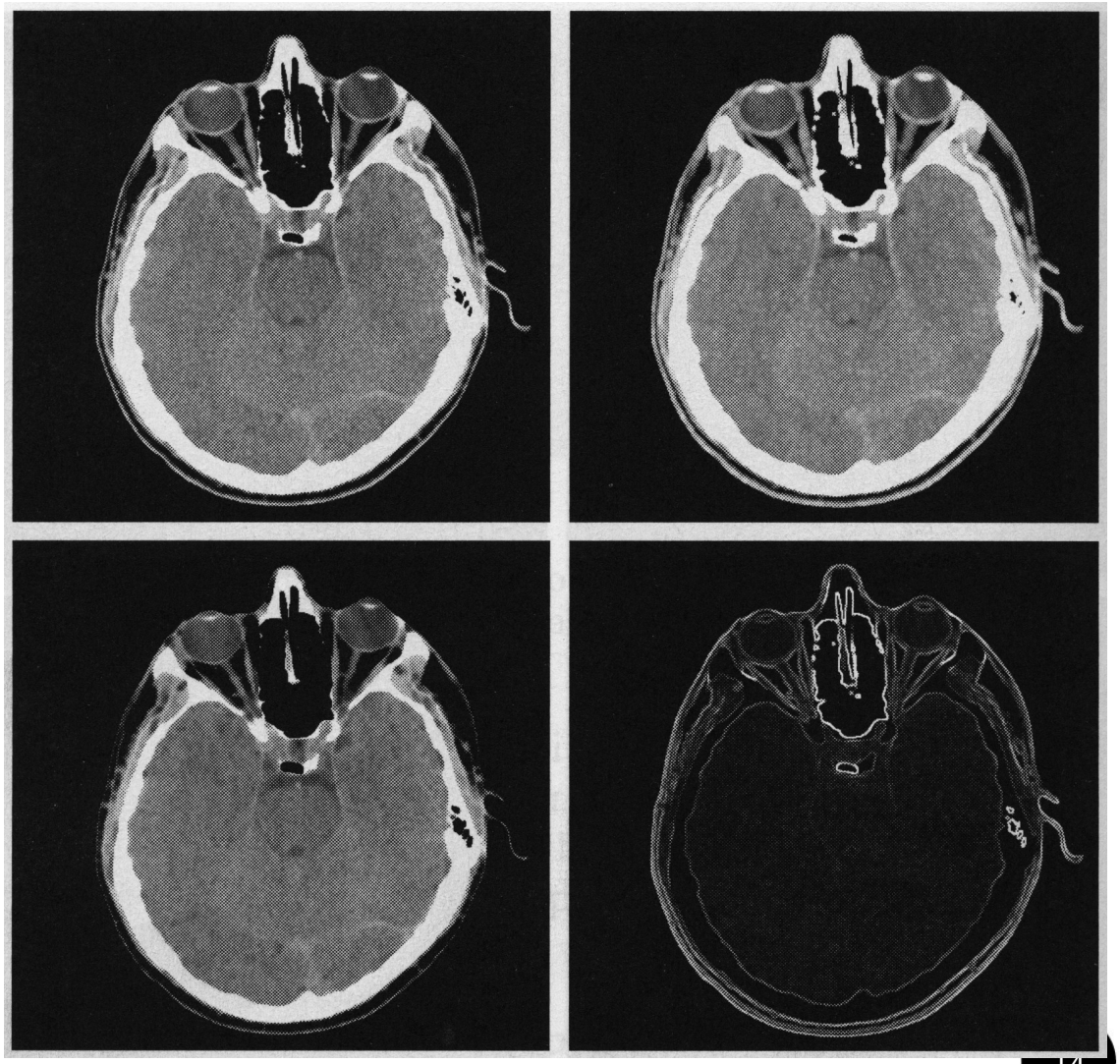
(c) (d)

- (a) 512 x 512 image of a head CT scan

  (b) Dilation

  (c) Erosion

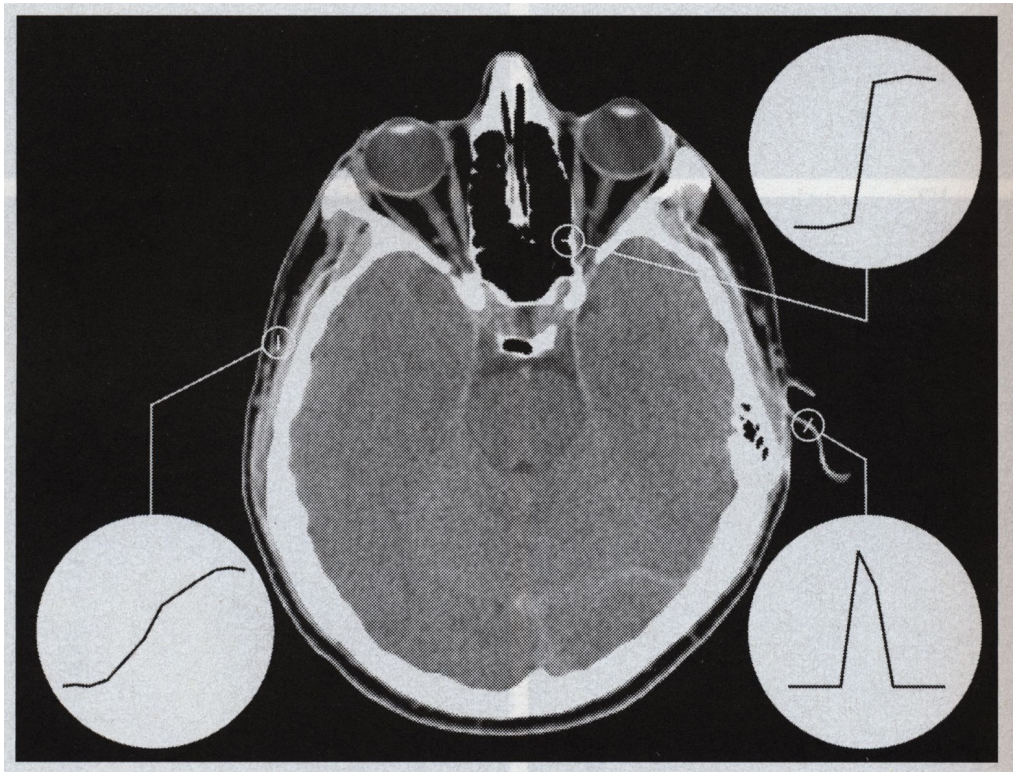
  (d) Morphologic gradient computed as the difference between (b) and (c)



(Gonzales, Woods)

Morphologic gradient

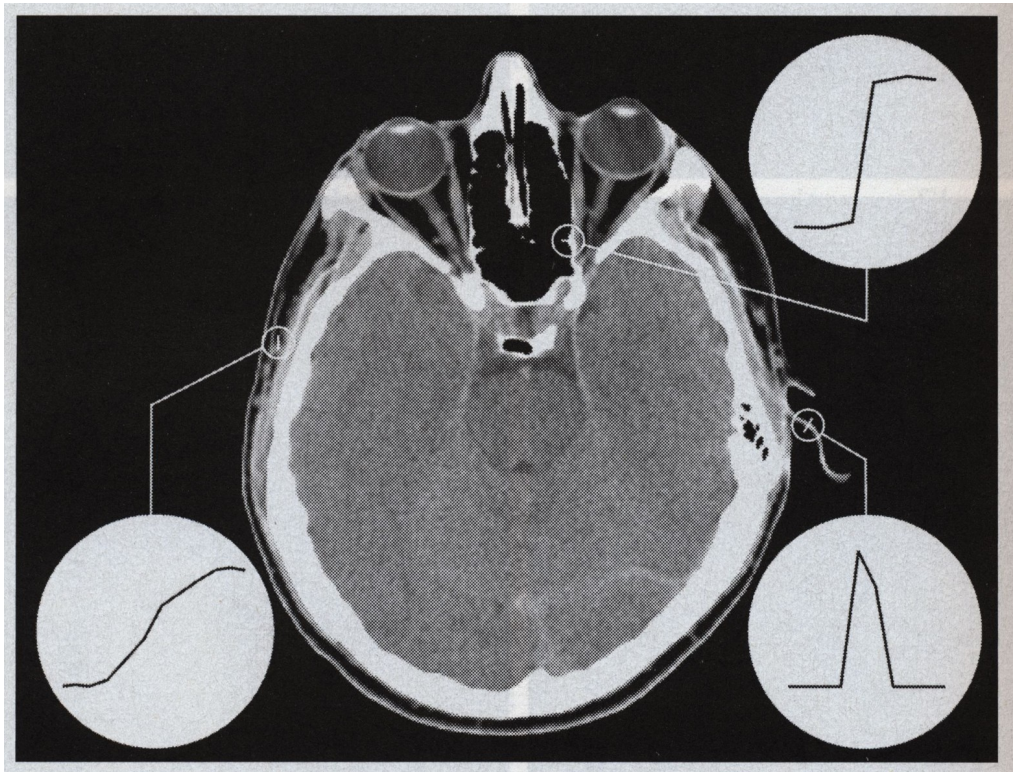
- Exercises 2:
  - Detecting contours of human organs in CT images using gray-scale morphological algorithms



(Gonzales, Woods)

Morphologic gradient

- Exercises 2:
  - Detecting contours of human organs in CT images using gray-scale morphological algorithms (hint: link edges between image slices using 24-connectivity)



(Gonzales, Woods)

Top-hat and bottom-hat transformations

- The top-hat transformation of a gray-scale image f is defined as f minus its opening:

$$T_{\text{hat}}(f) = f - (f \circ b)$$

- The bottom-hat transformation of f is defined as the closing of f minus f:

$$B_{\text{hat}}(f) = (f \bullet b) - f$$

- Principal applications of these transformations is in extracting objects from an image by using a structuring element in the opening or closing operation that does not fit to those objects that need to be extracted. The difference operation than yields an image in which only the extracted objects remain.
- The top-hat transformation is used for light objects on a dark background
- The bottom-hat transformation is used for dark objects on a light background
- An important use of top-hat transformations is in correcting the effects of nonuniform illumination

  → Proper (uniform) illumination plays a central role in the process of extracting objects from the background (i.e., in *segmentation of images*)

Top-hat and bottom-hat transformations

- Shading correction,

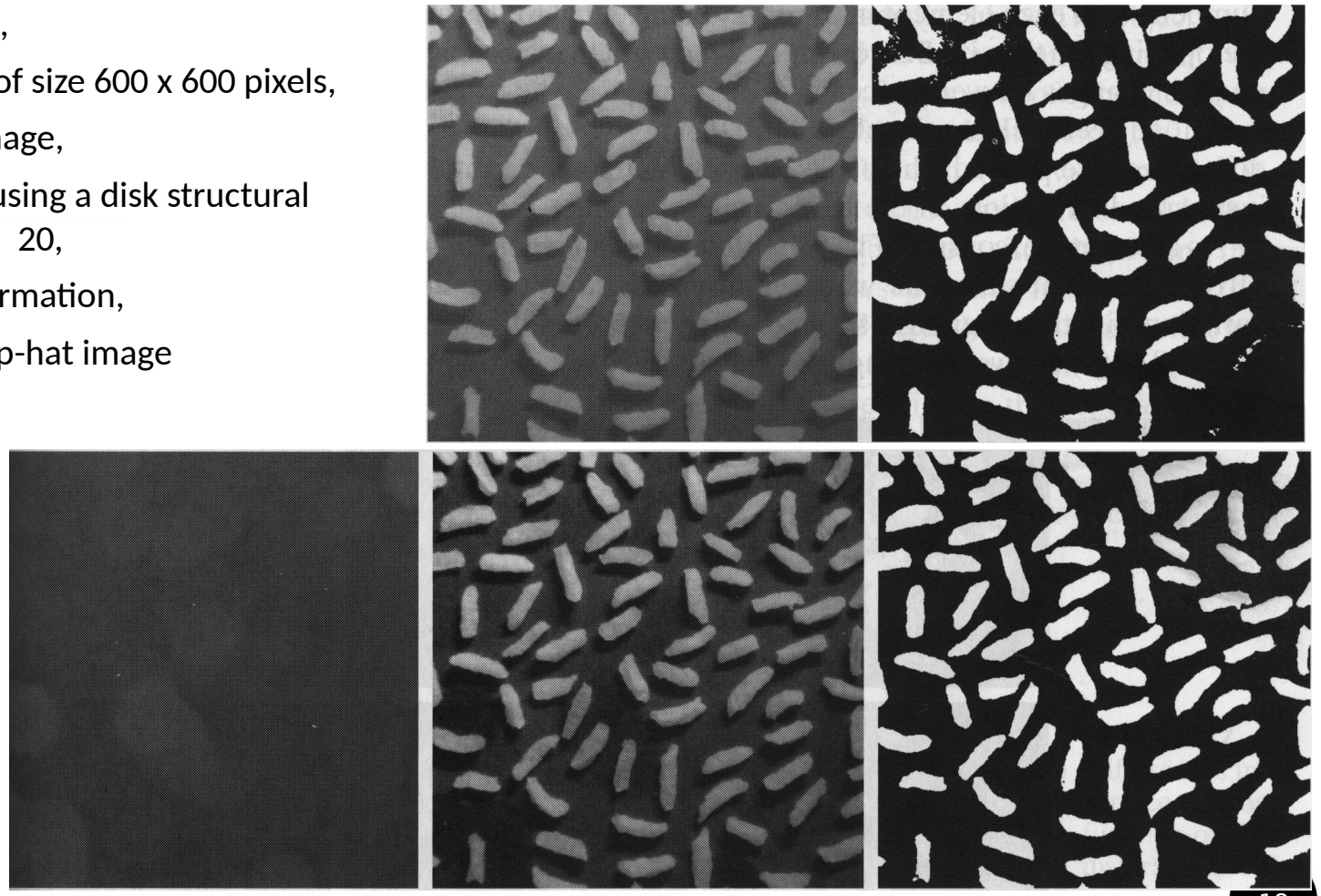
  (a) Original image of size 600 x 600 pixels,

  (b) Thresholded image,

  (c) Image opened using a disk structural element of radii 20,

  (d) Top-hat transformation,

  (e) Thresholded top-hat image



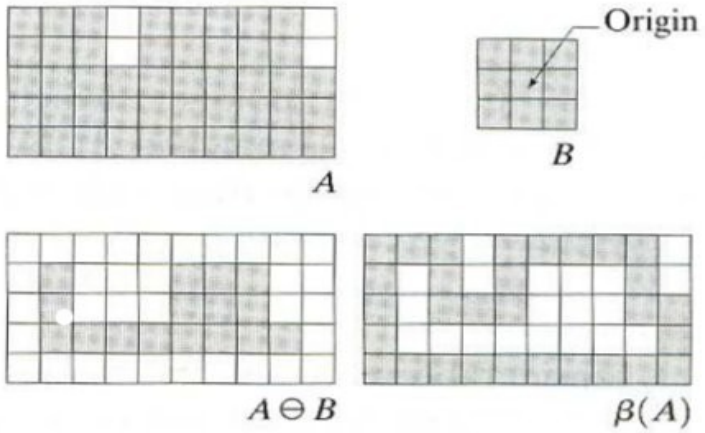
(Gonzales, Woods)

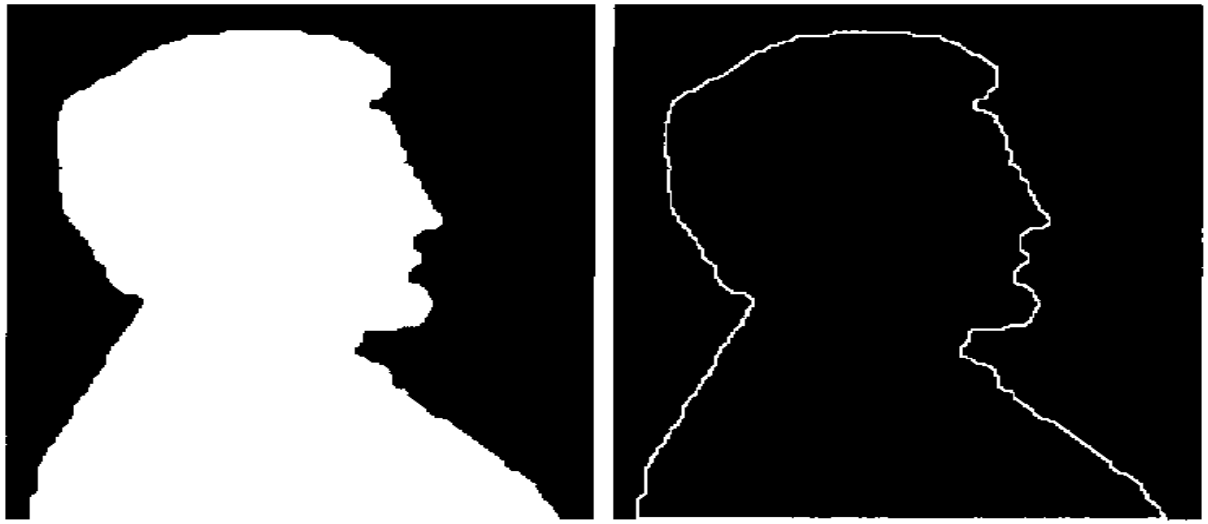
Boundary extraction

- Boundary extraction

  The boundary of a set A can be obtained by first eroding A by B (a suitable structuring element) and then performing the set difference between A and its erosion

$$\beta(A) = A - (A \ominus B)$$





(Gonzales, Woods)

Top-hat transformation and boundary extraction

- Detecting flecks/drusen in OCT retinal images using top-hat transformation (left, right), detecting geographic atrophy in OCT retinal images using boundary extraction (left)